EXTRAPOLATION TO THE 50-MB. LEVEL FROM 100-MB. DATA IN ANTARCTICA*

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ABSTRACT

Winter-month data obtained at United States Antarctic stations during the IGY are used to compute coefficients for regression equations relating the 100-mb. temperatures and heights to the 50-mb. heights. It is found upon trial that the layer thickness can be estimated as accurately from the 100-mb. temperature alone as from both the temperature and 100-mb. height; therefore, the simpler equations in only one variable are used. The average errors in computed heights are less than 2 dekameters and the extreme errors 6 dkm. or less when tests are performed on independent data.

A measure of the average lapse rates within the layer between 100 mb. and 50 mb., derived from the same data as the equations mentioned above, proves to be as good a predictor as any of the regression equations. These mean lapse rates show a regular variation with both latitude and month. The largest values (greatest instability) occur in the coldest months and the highest latitudes, and the greatest layer stability in lower latitudes and warmer months.

Additional 50-mb. heights are extrapolated from the observed 100-mb. data for all of the stations considered for radiosonde ascents reaching 100 mb. but not 50 mb.; means of the computed heights are compared with the means of the observed 50-mb. heights to determine if any bias in the observations can be demonstrated.

1. INTRODUCTION

During the International Geophysical Year in the Antarctic, regular radiosonde ascents at 12-hour intervals were made at, inter alia, the United States stations at the South Pole, Byrd, Little America, McMurdo Sound, Ellsworth, and Wilkes, and at the joint New Zealand-United States station at Hallett. In the colder six months of the year, April to September, and in particular during the four winter months from May to August, a large percentage of those ascents were terminated between 100 mb. and 50 mb., usually due to balloon bursts. The 50-mb. height data can be considerably augmented by an extrapolative technique to obtain reasonably accurate values of 50-mb. heights from the 100-mb. data.

The purpose of this investigation is to find the most feasible method of obtaining additional 50-mb. data by extrapolation from observed 100-mb. data, and to examine the thermal structure of the 100 mb. to 50 mb. layer and its variations with latitude and month over the Antarctic during the period considered.

2. EXTRAPOLATIVE METHODS

The first approach to the problem was to find coefficients for the regression equations relating the 50-mb. height, H_{50}, to the 100-mb. height, H_{100}, and the 100-mb. temperature, T_{100}. This is the same procedure followed by previous investigators in the Northern Hemisphere, as for example, Hering and Antanaitis [1], and currently in use by the U.S. Weather Bureau Stratospheric Analysis Project. Since in all cases we are to work from known 100-mb. data, the 50-mb. height is completely determined by the known 100-mb. height and the thickness, H_d, of the 100 mb. to 50 mb. layer. The original regression equations,

$$H_{50}=A_1H_{100}+A_2T_{100}+A_0 \tag{1}$$

may then be transformed to equations of the form

$$H_d=A_1'H_{100}+A_2T_{100}+A_0, \tag{2}$$

where $A_1'=A_1-1$.

The coefficients for equation (2) were computed for Little America for the six months April to September 1957 (228 ascents) to give a range through the period, and for the month of June 1957 for Little America, South Pole, Byrd and Wilkes (131 ascents) to give a range in latitude. The resulting A_1 coefficients showed a range of from -0.05 to $+0.015$, and none was significantly different from zero. Therefore, a new set of regression equations was obtained from this same data by computing the A_2' and A_0' coefficients for the general equation

$$H_d=A_2'T_{100}+A_0'. \tag{3}$$

A test of equations (2) and (3) on a limited amount of independent data (26 cases) resulted in an average error of 13 gpm. in the 50-mb. height using (2) and 11 gpm.

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TABLE 1.—*Mean values of thickness of 100 mb. to 50 mb. layer (H_d) and 100-mb. temperature (T_{100}) for winter months in the Antarctic. Thicknesses are in geopotential meters and temperatures in degrees Celsius.*

		South Pole (90° S.)	Byrd (80° S.)	Little America V—Ellsworth (78° S.)	Hallett (72° S.)	Wilkes (66° S.)	Monthly mean
April	H_d	----------	4291	4324	4384	4432	4343
	T_{100}	----------	−58.6	−57.5	−55.0	−53.1	−56.6
May	H_d	4064	4081	4122	4266	4350	4164
	T_{100}	−69.1	−68.3	−66.4	−59.6	−57.3	−64.6
June	H_d	3943	3977	4032	4114	4275	4050
	T_{100}	−74.8	−73.1	−71.1	−67.6	−60.4	−70.2
July	H_d	3852	3888	3896	4009	4038	3931
	T_{100}	−79.1	−77.4	−77.3	−72.2	−71.3	−75.7
August	H_d	3780	3821	3874	3946	4041	3880
	T_{100}	−83.4	−81.9	−79.3	−75.6	−71.9	−79.0
Sept.	H_d	3863	3902	3961	4113	4156	3988
	T_{100}	−81.0	−79.5	−76.5	−70.3	−69.1	−75.7
Mean (Lat.)	H_d	3969	3992	4033	4138	4214	(4058.26)
	T_{100}	−74.2	−73.2	−71.4	−66.8	−63.9	(−70.33)

TABLE 2.—*Computed lapse rate, °C./km., of 100 mb. to 50 mb. layer.*

	South Pole (90° S.)	Byrd (80° S.)	Little America V—Ellsworth (78° S.)	Hallett (72° S.)	Wilkes (66° S.)	Monthly mean
April	−1.5	−1.5	−1.2	−1.0	−0.8	−1.21
May	−1.9	−1.9	−1.8	−1.6	−0.7	−1.65
June	−2.1	−2.1	−1.7	−1.4	−1.0	−1.72
July	−2.2	−2.0	−2.0	−1.7	−1.5	−1.96
August	−1.9	−1.6	−1.6	−1.6	−1.1	−1.59
September	−1.0	−0.7	−0.8	−0.1	+0.3	−.79
Mean for station (Latitude)	−1.74	−1.66	−1.53	−1.25	−.79	−1.44

using (3). On the basis of this comparison, it was decided to use equations of the form (3) in computing regression coefficients for all stations except McMurdo Sound, and for all months of 1957 and 1958 (April to September) in which at least 8 ascents reached the 50-mb. level. The data from Ellsworth Station and Little America were combined, since these two stations are approximately the same latitude; a detailed listing of the data used is given by the parenthetical values in table 4 in the rows headed "Comp.". The McMurdo Sound data and unused data from the remaining stations were reserved for testing the results; table 3 gives a breakdown of the independent data by station and month.

In the course of the above computations, intermediate steps made it convenient to extract from the data the mean H_d and T_{100} for each station and month (table 1), and also to compute some of the correlation coefficients between these two parameters. The latter were generally quite high, ranging from +0.89 to +0.99 with a median value of those computed (18) of +0.95. Since the original computations were based on only 1455 cases in all, and individual station-months on from 8 to 43 ascents, it was thought advisable to test on independent data not only for the regression equations but also for equations based on the assumption of a constant lapse-rate of temperature within the layer.

If we assume that there is no significant amount of moisture within the layer from 100 mb. to 50 mb., then the mean virtual temperature and the mean temperature of the layer are approximately equal. The general equation which relates the thickness, H_d (in geopotential meters) and the mean virtual temperature, $\overline{T}_v$ (in ° K.) of a layer of air between two constant pressure surfaces p_1 (base) and p_2 (top) is

$$H_d = 67.442\ \overline{T}_v \log_{10} (p_1/p_2)$$

as given, for example, in [2]. We may now write for the 100 mb. to 50 mb. layer, substituting the mean temperature, $\overline{T}_k$, for the mean virtual temperature, the equation

$$H_d = 20.302\ \overline{T}_k. \tag{4}$$

If we now make the further assumptions that the lapse rate is constant with height within the layer and with time over the period of a month at a given station, then these lapse rates can be computed from the data given in table 1. The computed mean lapse rate, ($\overline{\Gamma}_d$), for each station and month, as well as the weighted mean lapse rates for each month and latitude, are given in table 2.

For extrapolation of the 50-mb. heights, we may now write a set of equations, based on the assumption of constant lapse rates, of the form

$$H_d = 20.302\left[273.16 + T_{100} + \frac{\Gamma_d H_d}{2000}\right] \tag{5}$$

This is the same as equation (4) with the term inside the brackets representing the mean absolute temperature of the layer for a given T_{100} (in °C.) if the assumption of constant lapse rate holds.

3. TESTS ON INDEPENDENT DATA

Independent data from the four months, May to August, were used to test four different methods of extrapolating the 50-mb. heights from the 100-mb. data, as follows:

Method I used equations of form (3) for a given latitude and month applied to data from the same latitude and month; Method II used equations of the form (3) for a given latitude, with the coefficients computed by combining the data from the four months, May to August, applied to data from the same latitude; Method III used equations of the form (3) for a given month, with the coefficients computed by combining all data for that month, applied to all independent data for the given month; Method IV was based on equations of the form (5), although to facilitate computations a nomogram and two graphs based on these equations, as described in the next section, were actually used.

The results of the testing of these various extrapolative

TABLE 3.—*Results of tests of four extrapolative methods on independent data.*

Station	Months	No. of cases	Mean errors (gpm.) in computed 50-mb. height			
			Method I	Method II	Method III	Method IV
		75°–90°S.				
Pole	May and Aug., 1958	15	8.8	15.7	18.0	10.7
Byrd	May to Aug., 1958	97	12.2	12.4	15.1	12.2
Ellsworth	May to Aug., 1957	28	11.8	10.7	13.6	14.0
McMurdo	May to Aug., 1957	165	14.8	12.7	14.7	12.8
Mean		(305)	13.4	12.5	14.9	12.6
		North of 75°S.				
Hallett	May, June and Aug. 1958.	74	22.5	23.1	25.3	22.6
Wilkes	May-Aug. 1958	79	19.8	(Independent for Method I only: not included in mean below)		
Mean		(379)	15.2	14.6	16.9	14.5

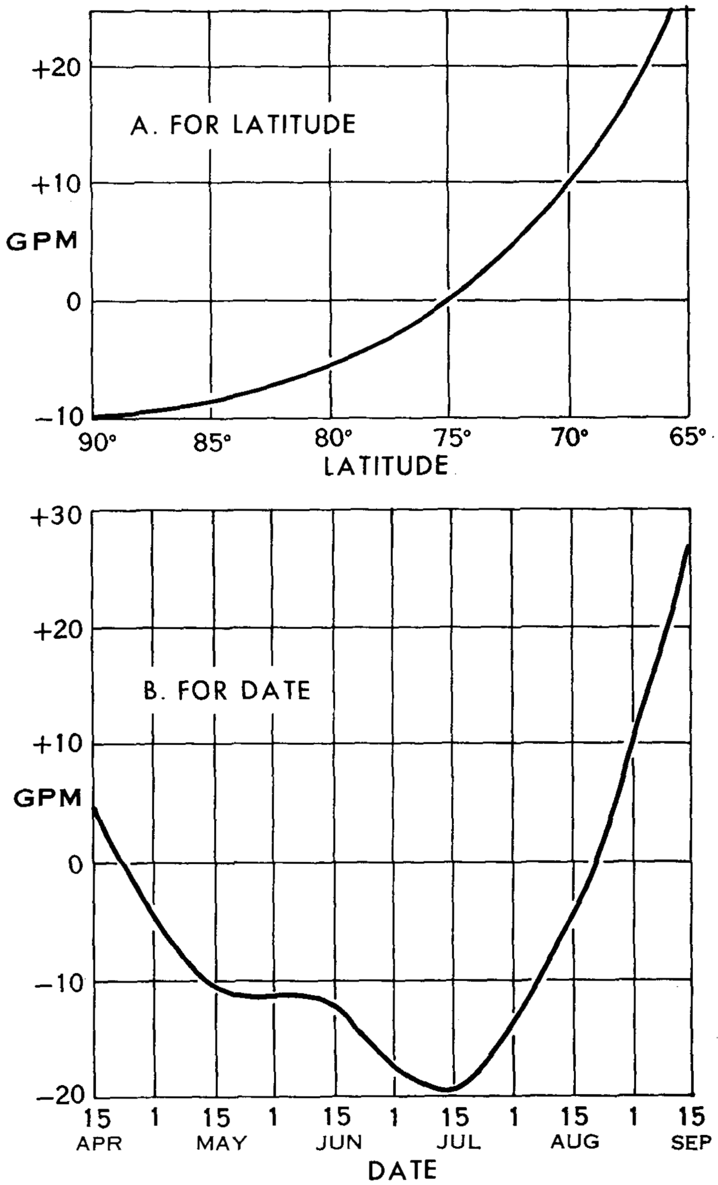

FIGURE 1.—Corrections to extrapolated 50-mb. height: (A) for latitude, (B) for date.

methods of obtaining the 50-mb. height are shown in table 3. The best results were obtained by using Method IV, although the small difference between this mean error and that from Method II hardly can be significant. In the area from latitude 77° S. to the Pole, the average errors using Method I, II, or IV were quite consistently between 10 and 15 gpm., while from Hallett northward more limited amounts of independent data indicated an average error using the best methods of 20 to 25 gpm. In general, the extreme error was about four times the average error.

4. CONSTRUCTION OF NOMOGRAM AND GRAPHS

When we substitute in equation (5) the values for the layer thickness H_d averaged over all stations for all of the 6 months, 4058.26 gpm., and the overall average Γ_d, $-1.44°$ C./km., it becomes

$$H_d = 5486.4 + 20.302\,T_{100},$$

where H_d is in geopotential meters and T_{100} in °C. For a given 100-mb. height H_{100}, a first approximation to the 50-mb. height H_{50} can be obtained, based on the mean overall lapse rate and layer thickness, from

$$H_{50} \approx H_{100} + 20.302\,T_{100} + 5486.4; \qquad (6)$$

this equation (6) was used to construct a nomogram, using H_{100} as ordinate and T_{100} as abscissa, from which the approximate H_{50} could be read off directly. The curves in figure 1 show the correction (gpm.) which must be applied to the nomogram value for latitude (fig. 1A) and for date (fig. 1B) to get the best estimate of the 50-mb. height using Method IV. Since the nomogram itself is a simple linear one it is not shown here.

5. LAPSE-RATE VARIATIONS

The correction curves of figure 1 are also graphs of the deviations of the layer lapse-rate from the overall mean value of $-1.44°$ C./km., with a correction of 10 gpm. on the ordinate scale the equivalent of a difference in lapse rate of 0.24° C./km. from the mean. Viewed in this light, figure 1A indicates that the stability of the 100 mb. to 50 mb. layer, averaged over the 6 months, April to September, increases in regular fashion from the Pole northward to the Antarctic Circle. Figure 1B shows that in general the layer stability averaged over all latitudes shows a gradual decrease from April until mid-July, and a comparatively rapid increase from then through September.

Comparison with the figures in table 1 shows that while the layer is cooling until mid-July, the stability is decreasing; this means that during this period the 50-mb. temperature is lowering more rapidly than the 100-mb. temperature, as already noted by Wexler [3]. The more rapid cooling at 50 mb. has been explained in terms of the vertical distribution of the atmospheric gases important in long-wave absorption, by Moreland [4], whose investigation indicates a maximum rate of cooling in this layer near 18 km., which is near the 50-mb. winter level.

The flattening of the curve in figure 1B in early June indicates that at this time, the temperature is falling at about the same rate at 100 mb. and 50 mb. Any explanation of this somewhat anomalous behavior is beyond the scope of this paper, although it might be noted in passing that June temperatures during 1957 and 1958 have been anomalous at other atmospheric levels, as demonstrated, for example, in Wexler's [3] study in the section on the "kernlose" Antarctic winter surface temperature pattern.

Finally, it might be well to note that for all of the 6 months, April to September, over the whole range of latitude the temperature falls with height in the 100 mb. to 50 mb. layer, with the exception of the lowest latitude station during the month with most sun (Wilkes, September), when an inversion began in this layer.

TABLE 4.—*Mean observed and computed 50-mb. heights. Heights in geopotential meters minus 18,000. Parenthetical figures are number of ascents used in determining means. Station-months with fewer than 8 ascents reaching 50 mb. not included*

Station		April	May	June	July	August	September
Pole	Year(s)	----------	1957	1957, 1958	1957, 1958	1957	1957, 1958
	Obs.	----------	908 (30)	640 (61)	388 (34)	308 (10)	324 (29)
	Comp.	----------	952 (17)	627 (48)	264 (68)	313 (37)	289 (48)
	Mean	----------	924 (47)	634 (109)	305 (102)	312 (47)	302 (77)
Byrd	Year(s)	1957	1957	1957	1957	1957	1957
	Obs.	1455 (8)	1004 (20)	859 (29)	448 (24)	416 (27)	457 (17)
	Comp.	1453 (11)	1104 (17)	864 (21)	372 (25)	440 (18)	411 (20)
	Mean	1454 (19)	1050 (37)	861 (50)	411 (49)	426 (45)	432 (37)
Ellsworth	Year(s)	1957, 1958	1958	----------	----------	----------	1957
	Obs.	1569 (52)	1038 (34)	----------	----------	----------	660 (16)
	Comp.	1690 (10)	984 (13)	----------	----------	----------	634 (31)
	Mean	1578 (62)	1023 (47)	----------	----------	----------	643 (47)
Little America V	Year(s)	1957, 1958	1957, 1958	1957, 1958	1957, 1958	1957, 1958	1957, 1958
	Obs.	1661 (71)	1206 (56)	933 (73)	489 (76)	424 (61)	483 (79)
	Comp.	1709 (22)	1374 (34)	942 (18)	455 (31)	416 (28)	431 (25)
	Mean	1672 (93)	1269 (90)	935 (91)	479 (107)	421 (89)	471 (104)
Hallett	Year(s)	1957	1957	1957	----------	1957	1957
	Obs.	1760 (22)	1587 (21)	1138 (13)	----------	656 (32)	899 (25)
	Comp.	1780 (5)	1511 (21)	1106 (18)	----------	569 (19)	900 (24)
	Mean	1764 (27)	1549 (42)	1119 (31)	----------	624 (51)	900 (49)
Wilkes	Year(s)	1957, 1958	1957, 1958	1957, 1958	1957, 1958	1957, 1958	1957, 1958
	Obs.	1957 (71)	1742 (63)	1657 (35)	1030 (38)	850 (35)	958 (57)
	Comp.	1939 (7)	1704 (8)	1642 (13)	942 (50)	797 (47)	933 (29)
	Mean	1956 (78)	1738 (71)	1653 (48)	980 (88)	820 (82)	950 (86)

6. COMPUTATION OF ADDITIONAL 50-MB. DATA

Table 4 shows the mean observed 50-mb. heights for several months and stations, and the mean computed height of the 50-mb. level from the radiosonde observations terminating between 100 mb. and 50 mb., using Method I for the computations. Method I is chosen here because it gave the best results of any of the four methods when applied to the same stations for which the coefficients were derived. It, therefore, seemed likely that Method I would give the best results when applied not only to the same station but to the very same month which gave the coefficients.

Table 4 also gives the weighted mean of the observed and the computed heights for each month and station for which computations were performed; i.e., those for which data were available and which had a minimum of 8 radiosonde observations reaching 50 mb. Numbers in parentheses following the mean heights are the number of 50-mb. heights, observed or computed, used in determining the mean. The heights are in geopotential meters, with 18,000 gpm. subtracted from each mean to save space.

Observers have noted that sounding balloons in the Antarctic, as elsewhere, tend to reach greater heights when rising through a relatively warm atmosphere than when rising through a relatively cold one. A priori, therefore, one might expect the mean of the computed heights during the Antarctic winter to be lower than that of the corresponding observed heights; however, the results tabulated in table 4 do not demonstrate conclusively any such bias, with the computed means lower than the observed about two-thirds of the time and higher the remaining one-third of the time.

7. CONCLUSIONS

Before listing conclusions to be drawn from this investigation, it is well to emphasize that only 7 stations' data from two winter seasons were available at the time of writing, and that the stations were concentrated in only about one-half of the continent. The effect of these limiting factors in modifying the conclusions listed is discussed in more detail in the results itemized below.

1. Extrapolated 50-mb. heights obtained using the regression equations with the 100-mb. temperature only as a predictor were as accurate as those obtained from equations using both the 100-mb. height and temperature.

2. Extrapolation of the 50-mb. heights using the assumption of constant lapse rates yields as good results as those of the computed regression equations.

Because of the limited amount of data, conclusions (1) and (2) above should be considered as true only for this set of data. It is entirely possible that the small superiority demonstrated by the constant-lapse-rate method and the single-predictor regression equations is due to chance, and that use of both the 100-mb. height and temperature as predictors might, in a large sample, show improvement over either of the other two methods. It is further possible that addition of different predictors might lead to a further refinement in accuracy; with the limited data it would in any case be impossible to demonstrate that any such improved accuracy was the result of the method's superiority rather than the result of chance, unless the new equations gave 50-mb. heights to a rather unlikely degree of accuracy.

3. The stability of the 100 mb. to 50 mb. layer was, in general, least when the mean temperature of the air was lowest; exceptions were noted in early June, when

the stability remained constant although the mean layer temperature decreased, and in August when some relative warming at 50 mb. increased the stability while the mean temperature was still falling slightly.

4. In the mean, there was a lapse of temperature in the 100 mb. to 50 mb. layer over the portion of the Antarctic considered from April through August, and in September only lower latitudes showed an inversion.

Strictly speaking, conclusions (3) and (4) are valid only for the specific locations and seasons considered. However, these conclusions can probably be extended safely to the continental area in which these stations are concentrated, although extension to the whole Antarctic area would depend on the unsafe assumption of approximate symmetry of temperatures about the geographic pole.

5. From extrapolative methods a considerable amount of additional 50-mb. height data accurate on the average to about 20 gpm. can be obtained for the winter months of the IGY at the Antarctic stations considered.

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REFERENCES

1. W. S. Hering and P. Antanaitis, "Specification of the 50-mb. Wind Field by Vertical Extrapolation," *GRD Research Notes* (Contributions to Stratospheric Meteorology), AFCRC-TN-58-450, ASTIA Document No. AD-152626. Geophysics Research Directorate, Air Force Cambridge Research Center, Cambridge, Mass., August 1958.
2. R. J. List (Ed.), *Smithsonian Meteorological Tables*, 6th Revised Edition, Smithsonian Institution, Washington, D.C., 1951 (p. 224)
3. H. Wexler, "Seasonal and Other Temperature Changes in the Antarctic Atmosphere," *Quarterly Journal of the Royal Meteorological Society*, vol. 85, No. 365, July 1959, pp. 196-208. Also (in Russian) in *Meteorologiîa i Gidrologiîa*, Leningrad, No. 3, March 1959, pp. 10-24.
4. W. B. Moreland, "Antarctic Stratospheric Thermal Structure, Circulation and Ozone Observations," paper delivered at *Symposium* on Antarctic Meteorology, Melbourne, Australia, February, 1959. (To be published.)

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